UNCLASSIFIED

AD 284 630

Reproduced
by the

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

63-1-1

284 630

MEMORANDUM
RM-3247-PR
SEPTEMBER 1962

SOME IMPLICATIONS OF THE EARTH'S GRAVITATIONAL FIELD FOR THE INTERNAL STRUCTURE OF THE EARTH

D. Oberste-Lehn

PREPARED FOR:
UNITED STATES AIR FORCE PROJECT RAND

The RAND Corporation

MEMORANDUM

RM-3247-PR

SEPTEMBER 1962

SOME IMPLICATIONS OF THE EARTH'S GRAVITATIONAL FIELD FOR THE INTERNAL STRUCTURE OF THE EARTH

D. Oberste-Lehn

This research is sponsored by the United States Air Force under Project RAND — Contract No. AF 49(638)-700 — monitored by the Directorate of Development Planning, Deputy Chief of Staff, Research and Technology, Hq USAF. Views or conclusions contained in this Memorandum should not be interpreted as representing the official opinion or policy of the United States Air Force. Permission to quote from or reproduce portions of this Memorandum must be obtained from The RAND Corporation.

The RAND Corporation

PREFACE

Irregularities in the earth's gravitational field affect the motions of bodies orbiting the earth. The purpose of this study is to determine the effect upon the earth's gravitational field of assumed variations in mass distribution between oceanic and continental areas. The results should be of interest to agencies and persons concerned with terrestrial structure and gravity, satellite orbits, and geodetic methods.

SUMMARY

The configuration of the earth's potential field has been determined from the orbits of near-earth satellites. This potential can be represented by a series expansion in spherical harmonics. The values of certain coefficients (J_n) for terms in this series have been calculated. The J_2 coefficient reflects the oblateness of the earth; the cause of the J_3 coefficient has not been definitely established.

The potential field can be expressed by surface gravity anomalies. Gravity anomalies result from inhomogeneities in mass distribution within a body. Such differences in mass distribution exist between oceanic and continental areas within the crust and upper mantle over large areas of the earth's surface. This Memorandum presents the results of an investigation of the gravitational fields of earth models performed to determine the nature of gravity anomalies to be expected from oceanic and continental areas. Using conventional crustal structure, the earth-model results indicate that a surface anomaly exists which is positive over continental blocks, and that its magnitude increases with increase in the assumed depth to isostatic compensation and with increase in positive topographic relief.

Application of the earth-model anomaly to the actual distribution of continents and oceans results in a calculated value for the gravitational anomaly which is of the same order of magnitude as, but of opposite sign to, that derived from satellite orbits. Conditions that might reconcile this discrepancy are presented.

ACKNOWLEDGMENTS

I wish to thank R. H. Frick and J. W. Kern for the benefit of their valuable discussions. The calculations were programmed for the IBM 7090 by Margaret E. Stickles.

CONTENTS

LIST OF SYMBOLS

a = distance from the center to the inner surface of a spherical shell

b = distance from the center to the outer surface of a spherical shell

G = gravitational constant ($6.673 \times 10^{-8} cm^3 g^{-1} sec^{-2}$)

g = gravitational attraction

g_n = gravity anomaly at earth's surface

Δg = gravity anomaly of earth models

J_n = coefficient

M = mass of the earth (5.975×10^{27}g)

P_n = Legendre polynomial

R = equatorial radius of the earth (6378.1 ± 0.1 km)

r = distance from center of body to the point where the value of potential or gravity is calculated

t = thickness, km

U = gravitational potential

θ = colatitude

ρ = average density

I. INTRODUCTION

The configuration of the earth's potential field, which relates directly to the gravitational field, has been determined from the orbits of near-earth satellites. In this study, the gravitational fields of certain earth models were investigated to determine how these fields would be affected by large areal variations in mass distribution within the earth's crust and upper mantle. The results obtained from the models were then compared with the satellite-derived data.

The gravitational potential of the earth may be expressed as an expansion in terms of spherical harmonics[1] as

$$U = \frac{GM}{R} \left\{ \frac{R}{r} - \sum_{n=2}^{\infty} J_n \left(\frac{R}{r} \right)^{n+1} P_n (\cos \theta) \right\}$$

where G is the gravitational constant; M, the mass of the earth; R, the earth's equatorial radius ($R = 6378.1 \pm 0.1$ km); r, the distance from the center of the earth to the point at which the potential is being calculated; P_n, the Legendre polynomial; and θ, the colatitude. The J_n coefficients determine the deviations of the gravitational potential from a spherical surface. The values of the J_n coefficients have been calculated from the orbits of near-earth satellites. Since the equatorial plane is chosen to pass through the earth's center of mass, J_1 is zero. Depending on the satellite data used and the method of calculation, the value of J_2 is about $(1082.79 \pm 0.15) \times 10^{-6}$,[1] and J_3 is about $(-2.4 \pm 0.3) \times 10^{-6}$.[1,2] The J_2 coefficient reflects the oblateness of the earth, which gives a flattening of about

1/298.2. The J_3 coefficient reflects a latitudinal asymmetry whose cause has not been definitely established. The form of the equation indicates that U is zero at infinity and that U increases in a positive direction as r decreases; this equation is valid for U exterior to the body whose potential is being calculated.

If the earth were a perfectly spherical body composed entirely of concentric spherical shells of uniform density, the gravitational attraction of the earth would be the same for any point on a spherical surface at a given distance exterior to the earth. Thus, for any reference sphere the value of gravity would be the same for any point on the sphere, and no anomalies would exist. But on and below the surface of the earth, areal variations in the density of materials are known to exist. In this case, values of gravity over the surface of the reference sphere vary; the corresponding deviations from an arbitrarily chosen mean value are called gravity anomalies.

According to Munk and MacDonald,(3) the J_n coefficients can also be expressed as gravity anomalies at the earth's surface by the formula

$$g_n = -(n - 1)\ (GMR^{-2})\ J_n \frac{P_n\ (\cos\ \theta)}{|P_n\ (\cos\ \theta)|}$$

Thus, corresponding to J_3

$$g_3 \approx -\ 2\ (985)\ J_3\ \text{cm sec}^{-2}$$

The amplitude of the third harmonic in the gravity-anomaly series is

$$g_3 = 4.8\ \text{mgl}^*$$

*One mgl = 0.001 gal; one gal = 1 cm sec^{-2}.

The interior of the earth consists of three major structural units: the core, the mantle, and the crust (Fig. 1). The core, which extends from the center of the earth to an average radius of approximately 3471 km, has an inner solid zone and an outer liquid zone. The mantle is solid, but due to high temperatures and pressures, it is capable of plastic flow over long time periods. It extends from the outer boundary of the core to a radius of from 6300 to 6360 km. The crust, extending from the top of the mantle to the surface (mean radius, 6371 km), is a relatively thin and rigid unit.

The density of the earth decreases with distance from its center. Major discontinuities occur between the core, mantle, and crust; lesser discontinuities are present at several other depths. Within the core and lower mantle, the density-layering is considered to approximate concentric spherical surfaces. Within the crust, and most probably within the upper mantle, the density pattern is irregular due to geological processes. A well-established and very distinct variation occurs in the density-layering of oceanic and continental crusts. The oceanic crust is considerably thinner than the continental crust, and it has a different composition (Fig. 2). From the surface downward, the oceanic crust is composed of a water layer ($\rho \approx 1.03$, $t \sim 5$ km), a sediment layer ($\rho \approx 2.32$, $t \sim 1$ km), and a basaltic layer ($\rho \approx 3.00$, $t \sim 5$ km). The total thickness of this crust averages about 11 km. The continental crust is composed chiefly of an upper granitic layer ($\rho \approx 2.67$) and a lower basaltic layer ($\rho \approx 3.00$). The total thickness of this crust varies considerably, as it may range from about 20 km under lowlands to about 50 - 60 km under large mountain ranges. The average thickness for continents is about 33 km.

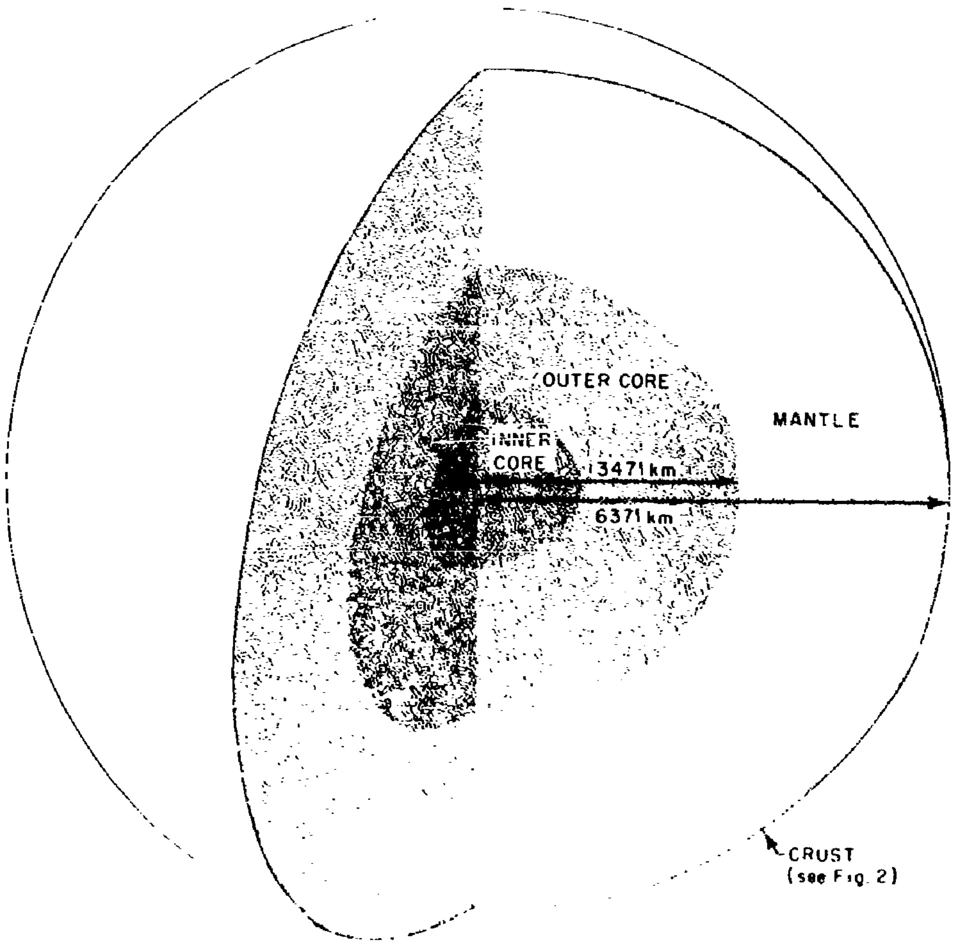

Fig. 1 — The internal structure of the earth

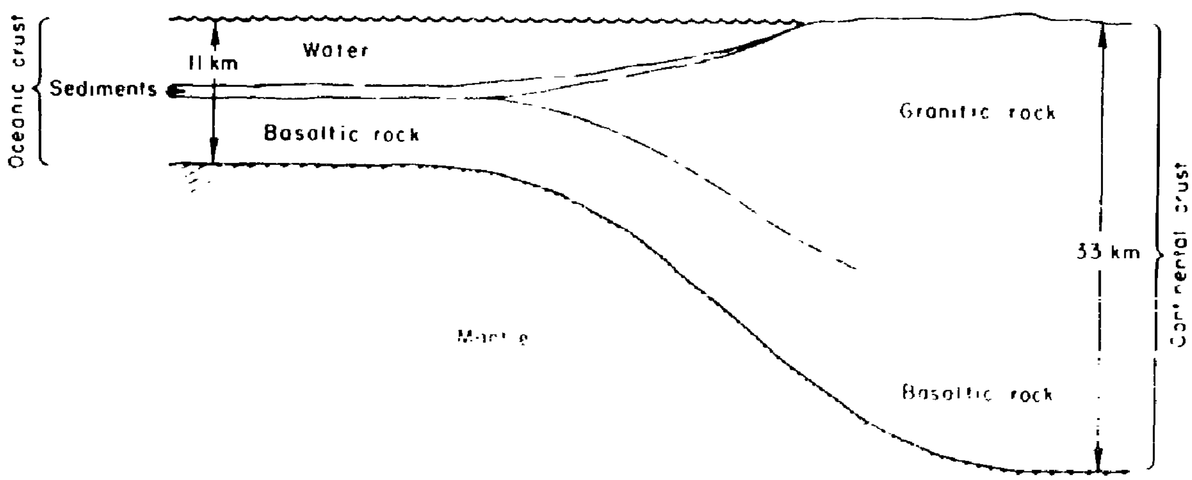

Fig 2—Schematic section showing the average thickness of oceanic and continental crusts

The preservation of these crustal differences is explained by the theory of isostasy. The earth is assumed to be in isostatic equilibrium, which means that at a "depth of compensation" D hydrostatic equilibrium prevails; thus at D any rock unit is under the same pressure, regardless of whether the unit is under mountain, lowland, or ocean. The depth of compensation is believed to occur as deep as, and probably below, the base of the crust.

The existence of isostasy is confirmed by gravity and geodetic measurements. An example of the maintenance of isostasy occurs during the formation of large deltas:* The geologic data show that the rock floor under the delta subsides gradually under the increasing load of deltaic sediments; however, the absence of large anomalies in gravity observations available in delta areas appears to indicate that the accretion of load on the subsided area is closely compensated by the subsidence rate.[4] Another example is the recoil of the areas unloaded by the melting of the Pleistocene ice sheets. Uplift motions are still continuing in the Scandinavian area and around the Great Lakes. Geodetic methods show that the middle part of the Gulf of Bothnia in Scandinavia has an uplift rate of 90 cm per 100 years. The associated gravity anomaly is decidedly negative, indicating that not all of the subcrustal mass that flowed outward during the glacial period has yet had time to move back.[4,5]

The densities and thicknesses of the various materials that compose the crust are determined by such tangible data as surface samples, drilled cores, and interpretations of seismic wave velocities. The degree of variability of these values is confined within rather narrow limits. As the depth from the surface increases, data become sparser and less reliable, so that corresponding uncertainties increase.

---

*Some controversy does exist as to the validity of this example as a demonstration of isostasy.

II. METHOD OF INVESTIGATION

In this study we are interested only in the gravity contrast contributed by differences in typical oceanic and continental mass distributions. Therefore, the effects on the earth's gravitational field due to the earth's rotation and oblateness, the ellipticity of the equator, and the attraction of other nearby bodies such as the sun and moon were excluded.

Spherical earth models incorporating either all-oceanic or all-continental layering were constructed, using a radius of 6371 km. Complete isostatic equilibrium was assumed. Density, gravity, and pressure values for the core and mantle are given in Table 1. The oceanic and continental sections, labeled O and C, respectively, in this report were based on typical sections used in studies on isostasy and the interior of the earth. The depth of compensation was varied to cover the range of likely possibilities for near-surface isostatic equilibrium.

An additional set of continental sections, C', having the high average topographic relief of 1 km was included. (One km, which is actually greater than the average relief of continental areas, was arbitrarily chosen as the outermost limit of possible large-scale variations due to topography.) Figure 3 shows the columnar representation of the density values and distribution of these sections.

The gravitational attractions of each of the resultant models were calculated using the formula

$$g = 4/3 \, \pi \, G \sum \rho \, \frac{(b^3 - a^3)}{r^2}$$

Table 1

DENSITY (AFTER BULLEN), GRAVITY, AND PRESSURE IN THE EARTH[5]

Depth (km)	Radius, r (km)	Density, ρ (g/cm^3)	Gravity, g (cm/sec^2)	Pressure, p (bars[a] x 10^6)
33	6338	3.32	983	0.009
80	6291	3.36	984	0.025
80	6291	3.87	984	0.025
200	6171	3.94	983	0.071
400	5971	4.06	981	0.149
600	5771	4.18	979	0.230
800	5571	4.30	977	0.313
1000	5371	4.41	975	0.398
1200	5171	4.52	974	0.485
1400	4971	6.63	975	0.574
1600	4771	4.74	977	0.666
1800	4571	4.84	980	0.759
2000	4371	4.94	987	0.855
2200	4171	5.03	996	0.954
2400	3971	5.13	1010	1.056
2600	3771	5.22	1028	1.161
2700	3671	5.27	1041	1.216
2900	3471	5.57	1068	1.330
2900	3471	9.74	1068	1.33
3000	3371	9.90	1048	1.41
3200	3171	10.20	1005	1.62
3400	2971	10.47	960	1.82
3600	2771	10.72	913	2.02
3800	2571	10.95	865	2.21
4000	2371	11.16	816	2.40
4200	2171	11.36	767	2.58
4400	1971	11.54	717	2.75
4600	1771	11.71	670	2.91
4800	1571	11.85	632	3.06
4982	1389	12.00	598	3.19
5121	1250	15.01	564	3.30
5400	971	16.16	457	3.53
5700	671	17.07	326	3.72
6000	371	17.65	184	3.85
6371	0	17.9	0	3.92

[a] The bar is the meteorologist's bar of 10^6 dynes/cm^2, which is equal to 0.987 atm. The pressure at the center is thus 3.92 x 10^{12} cgs units.

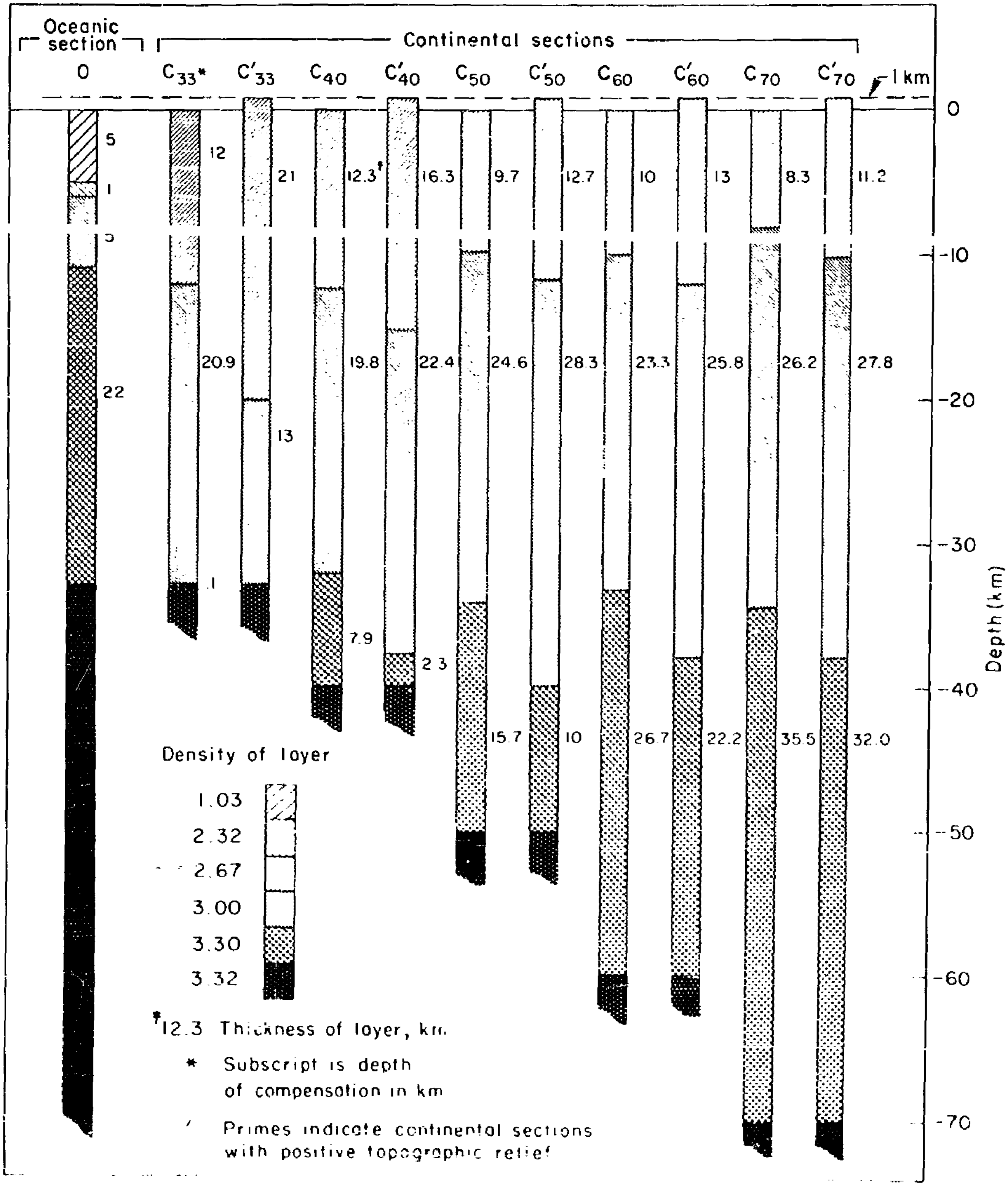

Fig. 3— Density versus depth for models of oceanic and continental sections

where g is the gravitational attraction; ρ, the average density; b and a, the distances from the center to the outer and inner surfaces, respectively, of spherical shells; and r, the distance from the center of the sphere to the point at which the attraction of gravity is calculated ($r \geq b$).

III. RESULTS AND DISCUSSION

With the value of the gravitational attraction for the oceanic section being used as a reference in defining the surface gravity anomalies ($g_o = 0$), the resulting anomalies, Δg, for the continental sections are shown in Table 2.

Table 2

CONTINENTAL GRAVITY ANOMALIES

C_D	Δg (mgl)	C'_D	Δg (mgl)
C_{33}	+3.7	C'_{33}	+4.7
C_{40}	+3.7	C'_{40}	+5.5
C_{50}	+4.2	C'_{50}	+31.0
C_{60}	+4.3	C'_{60}	+61.0
C_{70}	+4.9	C'_{70}	+89.0

The subscripts of C indicate the depth in km to compensation. For the C models, Δg is positive and increases slightly as depth to compensation increases. For the C' models, Δg is also positive, but, in contrast to the C models, is greater in magnitude and increases at a greater rate as depth of compensation increases. Outward from the surface of the models, the gravitational attraction decreases at the same rate for each set of models having the same depth of compensation.

The earth models indicate that the magnitudes of Δg are comparable to g_3 (4.8 mgl) for continental blocks of low average relief for depths

of compensation near to and greater than 70 km, and for continental blocks of high average relief at shallower depths of compensation. The former condition more closely approximates that of the real earth. The model values are expected to give only rough approximations of Δg because (1) isostatic equilibrium is not complete; (2) the earth is not a perfect sphere; (3) about 85 per cent of the earth's continental area has low relief (this high percentage is attributable, in part, to the fact that continental-shelf areas are considered part of the continental blocks); and (4) rock layers in the earth are not concentric shells of uniform density.

There seems to be a discrepancy, however, in the sign of the anomaly. Based on the earth-model anomalies (Table 2), positive gravity anomalies would occur over the midlatitudes of the Northern Hemisphere where the continental areas are concentrated. However, in the evaluation of the potential formula for $n = 3$, P_3 is negative in the midlatitudes of the Northern Hemisphere, causing the equipotential surfaces to deviate inward. Likewise, the sign of the gravity anomaly g_3 (see p. 2) for the same latitudes is also negative. Thus, whereas the model results suggest a deviation in the shape of the earth's potential roughly resembling an upside-down pear-shape, the third zonal harmonic, as derived from satellite orbits, gives a right-side-up pear-shape (stem at the North Pole). The pear-shape configuration refers to that of the potential, and not to the geometrical shape of the earth. The following explanations have been considered in trying to reconcile this discrepancy: misinterpretation of the satellite data; error in the structure of the standard sections; variations of density distribution deeper in the mantle either associated with, or independent of, the distribution of continents; or lack of isostatic equilibrium. Because there has been close agreement between

determinations of the J_3 coefficient made by several persons and from various satellite orbits, the configuration suggested by the third zonal harmonic is assumed to be correct.

Traditionally, the densities for typical oceanic and continental crustal sections cross one another at a depth equivalent to that of the base of the oceanic sediments. Above this depth, the density is greater for the continental section; below it, the density for the continents is always less, down to the depth of compensation. In a spherical body, this type of structure ensures a positive anomaly over the continental section. (Some of the columns in Fig. 3 show a short interval of equal density associated with the crossover.) By changing the structure of the typical sections so that the mass is reduced and the center of gravity is lowered underneath the continents, it may be possible to achieve a negative anomaly over the continents. This change can be accomplished by having two density crossovers, i.e., downward from the surface the continental-section density would read: more dense than the oceanic-section density, less dense, and more dense.

Variations of density deeper in the mantle could account for the satellite-observed potential configuration. If these variations are independent of the placement of continents, then some mechanism other than hydrostatic equilibrium is needed to explain them. If they are associated with continents, density variations would be postulated to occur at greater depths in the mantle, perhaps to 200 km, or even deeper. Models based on the latter supposition would be speculative.

If the requirement that the earth be in isostatic equilibrium were not strictly adhered to, standard sections could be retained by

postulating some sort of mass-deficiency in the mantle under the continents. This might be accounted for should crustal rigidity allow material to be eroded from continents and deposited into oceans at a rate greater than that at which the crust and upper mantle can readjust toward complete isostatic equilibrium. Treatment of this case by earth models would be difficult.

IV. CONCLUSIONS

Layered sequences of large areal extent having different mass distributions in isostatic equilibrium can cause surface gravity anomalies. Earth models using typical oceanic and continental sections result in a positive anomaly over continents. This anomaly would produce a potential configuration inverse to that indicated by the third zonal harmonic of the earth's potential field as derived from satellite orbits.

Assuming the interpretation based on satellite orbits to be correct, the above discrepancy might be accounted for by (1) error in the structure of the standard sections, (2) variations of mass distribution existing deeper in the mantle, perhaps on the order of 100 to 200 km, or more, or (3) isostatic equilibrium sufficiently incomplete to permit a mass deficiency to exist under the continents. The discrepancy seems to be most plausibly explained by variable mass distribution deeper within the mantle, with some contribution from incomplete isostatic equilibrium.

REFERENCES

1. King-Hele, D. G., "The Earth's Gravitational Potential, Deduced from the Orbits of Artificial Satellites," *J. Geophys. Royal Astronom. Soc.*, Vol. 4, 1961, pp. 3-16.

2. O'Keefe, J., A. Eckels, and R. K. Squires, "Vanguard Measurements Give Pear-shaped Component of Earth's Figure," *Science*, Vol. 129, 1959, pp. 565-566, *J. Geophys. Res.*, Vol. 64, 1959, p. 2389.

3. Munk, W. H., and G.J.F. Mac Donald, "Continentality and the Gravitational Field of the Earth," *J. Geophys. Res.*, Vol. 65, 1960, pp. 2169-2172.

4. Gutenberg, B., ed., *Internal Constitution of the Earth*, 2d. ed., Dover Publications, Inc., New York, 1951.

5. Heiskanen, W. A., and F. A. Vening Meinesz, *The Earth and Its Gravity Field*, McGraw-Hill Book Company, New York, 1958.

UNCLASSIFIED

UNCLASSIFIED